647510 **BSc5MatC501x** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination

December -2020 (NEW COURSE)

Mathematical Analysis -1 & Abstract Algebra -1(Core)

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

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1 (A) Answer any two out of three. **[10]**

1. X is metric space and E $\subset$ X. a is limit point of E. Prove that There exist infinite points of E in every neighborhood of a.
2. X is metric space and E $\subset$ X. Show that E' is closed.
3. Explain cantor set and define it. Show that cantor set is closed.

(B) Answer any one out of two. **[04]**

1. d is define on R as; for $\forall$ x,y $\in$ R

   $$d(x,y)\begin{cases} = 0 & ; \quad x = y \\ = \sqrt{(x^2 + y^2)}; & x \neq y \end{cases}$$ Then show that (R, d) is metric space.

2. find E^0 where E=[2,7].

2 (A) Answer any two out of three. **[10]**

1. Function f is bounded in [a,b].P and P^* be two partitions of [a,b]

   Such that P $\subset$ P^* then show that $U(P^*,f) \leq U(P,f)$.
2. if functions f and g are R-integrable in [a,b] then show that f+g is also R-integrable in [a,b] and

   $$\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$
3. using darbour's theorem find value of $\lim\limits_{n\to\infty}\left[+\frac{n^n}{n!}\right]^{\frac{1}{n}}$

(B) Answer any one out of two. **[04]**

1. Find $\int_1^2 (2x + 1)dx$ using definition of R-integration.
2. If function f is continuousin [a,b] then show that f is R-integrable in [a,b].

3 (A) Answer any two out of three. **[10]**

1. State and prove fundamental theorem of integration.
2. State and prove general form of first mean value theorem For R-integration.
3. G=R-{1}. * is defined on G as ; for $\forall$ a,b $\in$ G , a*b=a+b-ab.

   Show that * is binary operation on set G and also show that (G,*) is a group.

(B) Answer any one out of two. **[04]**

1. Show that $\int_0^{\frac{\pi}{4}} x tan(x) dx < \frac{\pi}{8} \log_e 2$.
2. Show that in group G, for $\forall$ a,b $\in$ G, equation a*x=b has unique solution in G.

4 (A) Answer any two out of three. **[10]**

1. State and prove Lagranges theorem.
2. Prove that every permutation f of finite set is a product of disjoint cycles.
3. Define alternating group and prepare group table for Alternating group of symmetric group S_3 in usual Notation.

(B) Answer any one out of two. **[04]**

1. G is finite group. For a,b $\in$ G Show that $O(b^{-1}ab)=O(a)$.
2. $\sigma \epsilon$ S_6 where $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 2 & 1 & 3 & 5 \end{pmatrix}$ then find σ^{-1} and o(σ). Also check whether σ is even or odd.

5 (A) Answer any two out of three. **[10]**

1. Prove that infinite cyclic group has exactly two generators.
2. State and prove Cayley theorem.
3. if $(G,*) \cong (G',*')$. Show that G is commutative iff G' is commutative. Where G and G' are groups.

(B) Answer any one out of two. **[04]**

1. G is group and H≤G. if index of H in G is 2 then prove that H is Normal Subgroup of G.
2. Prepare group table in usual notation for quotient group Z_6/<3>. Where $(Z_6,+_6)$ is group.

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